HEATING OF A MICROPOLAR LIQUID DUE TO VISCOUS ENERGY DISSIPATION IN CHANNELS. II. COUETTE FLOW

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The heating of a micropolar liquid due to viscous energy dissipation as it flows in a plane channel when one of the plates moves at a constant velocity relative to the other is investigated.

The heating of a micropolar liquid (MPL) due to viscous energy dissipation during Poiseuille flow in a plane channel was considered in the first part of the present article [1]. The investigation of the dissipative heating of an MPL in the case of Couette flow is of particular interest, since it is just flows close to this type which occur in a number of cases in practice. For example, under certain conditions the flow of a lubricant or the behavior of magnetic-liquid seals can be described by Couette flows in a first approximation.

Below we investigate the heating of an MPL due to viscous energy dissipation as it flows in a plane channel when one of the plates moves at a constant velocity relative to the other.

As in the previous case, suppose that the liquid is incompressible, its physical properties are constant, mass forces and their moments can be neglected, and hydrodynamics and heat exchange are stabilized.

We match the x axis of the Cartesian coordinate system with the upper face of the lower stationary plate of thickness H_2. The upper plate of thickness H_1, located at a distance h from the lower one, moves at a constant velocity $\vec{V}$ in the direction of the x axis. As earlier, we analyze the hydrodynamic problem independently of the thermal problem, i.e., we assume that dissipative heating of the liquid has little effect on its properties.

Under the adopted assumptions, the system of differential equations for the nonzero components of the velocity and microrotation vectors, $v_x(y)$ and $\nu_z(y)$ [2], is written in the form

$$(\mu+\varkappa)\frac{d^2v_x}{dy^2}+\varkappa\frac{d\nu_z}{dy}=0, \tag{1}$$

$$\gamma\frac{d^2\nu_z}{dy^2}-\varkappa\frac{dv_x}{dy}-2\varkappa\nu_z=0. \tag{2}$$

Solving it with the boundary conditions

$$v_x(0)=0,\ v_x(h)=V,\ \nu_z(0)=-\frac{\alpha}{2}\left(\frac{dv_x}{dy}\right)_{y=0},\ \nu_z(h)=-\frac{\alpha}{2}\left(\frac{dv_x}{dy}\right)_{y=h}, \tag{3}$$

we arrive at the expressions for v_x and ν_z,

$$v_x(\tilde{y})=\frac{V}{L}[N(1-\mathrm{ch}k\tilde{y})-\mathrm{sh}k\tilde{y}+kP\tilde{y}], \tag{4}$$

$$\nu_z(\tilde{y})=\frac{kVW}{2hL}\left(\mathrm{ch}k\tilde{y}+N\mathrm{sh}k\tilde{y}-\frac{P}{W}\right), \tag{5}$$

where

$$\tilde{y}=\frac{y}{h},\ L=kP+2N,\ k=\bar{k}h=\left(\frac{2\mu+\varkappa}{\mu+\varkappa}\frac{\varkappa}{\gamma}\right)^{1/2}h,\ N=\frac{1-\mathrm{ch}k}{\mathrm{sh}k},$$

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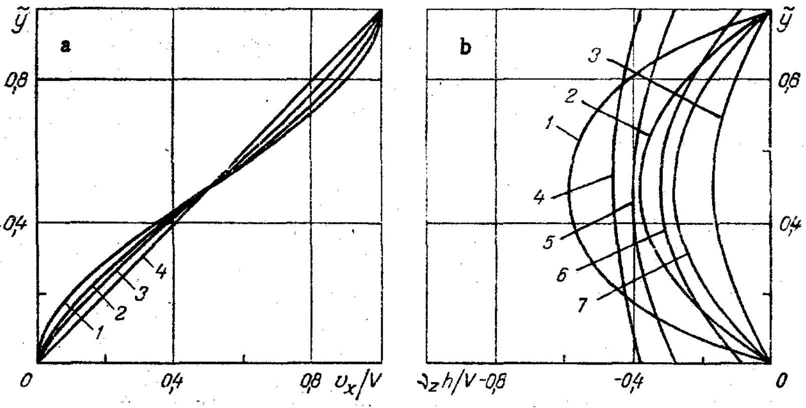

Fig. 1. Velocity v_x/V (a) and microrotation $\nu_z h/V$ (b) in the channel: a) k = 2.5, ξ = 1, $\delta_0 = 10^4$ (1); 10 (2); 2 (3); 0 (4); b) δ_0 = 5, ξ = 1, k = 5 (1); 2 (2); 1 (3); δ_0 = 2, k = 2, ξ = 0.2 (4); 0.4 (5); 0.8 (6); 1 (7).

$$P = 1 + \frac{2}{\xi\delta_0},\ W = 1 + \frac{2}{\delta_0},\ \xi = 1 - \alpha,\ \delta_0 = \frac{\varkappa}{\mu^N},\ \mu^N = \mu + \frac{\varkappa}{2}.$$

Curves of v_x/V and $\nu_z h/V$ constructed from (4) and (5) are shown in Fig. 1. Allowance for the internal microstructure of the liquid when calculating the velocity v_x results in its increase in the half of the channel cross section adjacent to the moving wall and in its decrease in the other half (Fig. 1a). A numerical analysis of Eq. (4) shows that in this case the variation of v_x/V as a function of h has a maximum corresponding to the value k ≈ 2.5, and also grows as ξ and δ_0 increase. For $\delta_0 < 1$ the variation of v_x/V in the region of $0.1 \leqslant \tilde{y} \leqslant 0.9$ does not exceed 5% for any k and ξ. At the same time, $\nu_z h/V$ has a monotonic dependence on k, as follows from the curves for the microrotation presented in Fig. 1b. It is seen that the smaller h for constant δ_0, $\tilde{k}$, and ξ or the larger ξ for constant δ_0, $\tilde{k}$, and h, the more the microrotation differs from $\vec{w}$, i.e., the more strongly the micropolarity of the liquid is manifested.

In the case under consideration the expression for the viscous-energy-dissipation function is similar to that used in the first part of the present article:

$$\Phi m(y) = (\mu + \varkappa)\left(\frac{dv_x}{dy}\right)^2 + 2\varkappa\left(\frac{dv_x}{dy} + \nu_z\right)\nu_z + \gamma\left(\frac{d\nu_z}{dy}\right)^2. \tag{6}$$

Substituting the expressions for the velocity (4) and microrotation (5) into (6), we obtain

$$\Phi m(\tilde{y}) = \Phi^N \frac{k^2}{L^2}\left[W(1 + N^2)\,\mathrm{ch}\,2k\tilde{y} + 4WN\,\mathrm{ch}k\tilde{y}\,\mathrm{sh}k\tilde{y} - 2P(\mathrm{ch}\,k\tilde{y} + N\mathrm{sh}k\tilde{y}) + P^2\right], \tag{7}$$

where $\Phi^N = V^2\mu^N/h^2$ is the viscous-energy-dissipation function for a Newtonian liquid with a shear viscosity μ^N.

Let the constant temperature T_w of the outer surfaces of the channel plates be assigned. We formulate the problem of the heating of an MPL with a distribution function of dissipative heat sources (7) as follows:

$$\lambda_2 \frac{d^2T_2}{dy^2} = -\Phi^m(y),\ \lambda_1 \frac{d^2T_1}{dy^2} = 0,\ \lambda_3 \frac{d^2T_3}{dy} = 0, \tag{8}$$

$$\lambda_2 \left.\frac{dT_2}{dy}\right|_{y=h} = \lambda_1 \left.\frac{dT_1}{dy}\right|_{y=h},\ \lambda_2 \left.\frac{dT_2}{dy}\right|_{y=0} = \lambda_3 \left.\frac{dT_3}{dy}\right|_{y=0},\ T_1(h + H_1) = T_w,\ T_3(-H_2) = T_w,\ T_1(h) = T_2(h),\ T_2(0) = T_3(0). \tag{9}$$

Here the indices 1, 2, and 3 indicate the temperature distributions in the moving plate, the liquid, and the fixed plate, respectively.

Integrating the system of equations (8) with the boundary conditions (9), after a series of transformations we obtain an expression for the temperature distribution in the liquid,

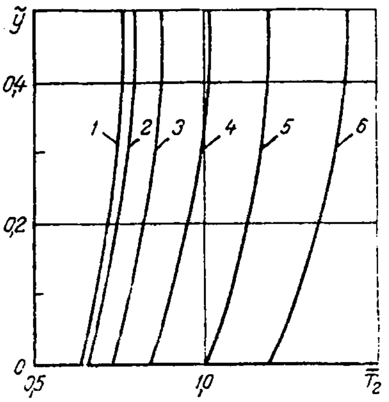

Fig. 2.

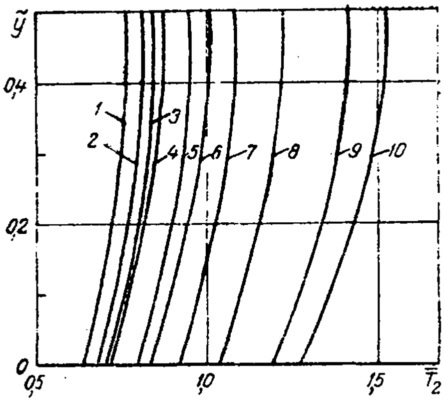

Fig. 3.

Fig. 2. Temperature field in a channel with $\lambda_s/\lambda_l = 78$, $l = 100$, $k = 1$, $\delta_0 = 2$, and $\xi \to 0$ (1): $\xi = 0.2$ (2); 0.4 (3); 0.6 (4); 0.8 (5); 1 (6).

Fig. 3. Temperature field in MPL ($\delta_0 = 2$, $\xi = 1$) in different channels with $k = 10$ (3); 5 (5); 4 (6); 3 (7); 2 (8); 1 (9); 0.1 (10) and in water ($k = 7$, $\delta_0\xi = 1.45$) with $\delta_0 = 7.25$, $\xi = 0.2$ (2) and $\delta_0 = 1.45$, $\xi = 1$ (4); curve 1 corresponds to a Newtonian liquid ($\delta_0 \to 0$). $\lambda_s/\lambda_l = 78$, $l = 100$.

$$T_2(\tilde{y}) = \frac{1}{L^2}\left[2P(\mathrm{ch}k\tilde{y} + N\mathrm{sh}k\tilde{y}) + kWN\tilde{y} + \frac{WN}{2}\mathrm{sh}\,2k\tilde{y} - \frac{W(1+N^2)}{4}\mathrm{ch}\,2k\tilde{y}\right] - \frac{1}{2}\left(\frac{kP\tilde{y}}{L}\right)^2 + C_1\tilde{y} + C_2, \quad (10)$$

where

$$C_1 = \left(\frac{k}{L}\right)^2\left[\frac{\frac{\lambda_s}{\lambda_l}(Q - S - Z) - l_2\left(\frac{2PN}{k} + S\right)}{l_1 + l_2 + \frac{\lambda_s}{\lambda_l}} + S\right];$$

$$C_2 = \frac{\lambda_l}{\lambda_s}\left(\frac{2PNk}{L^2} - C_1\right)l_2 + Z\left(\frac{k}{L}\right)^2;$$

$$S = \frac{1}{k}\left[\frac{W(1+N^2)\,\mathrm{sh}\,2k}{2} + 2WN\,\mathrm{sh}^2k - 2PN\mathrm{ch}k - 2P\mathrm{sh}k + kP^2\right];$$

$$Q = \frac{1}{k^2}\left[\frac{W(1+N^2)\,\mathrm{ch}\,2k}{4} + \frac{WN\,\mathrm{sh}\,2k}{2} - 2P\mathrm{ch}k - 2PN\mathrm{sh}k - kWN + \frac{k^2P^2}{2}\right];$$

$$Z = \frac{1}{k^2}\left[\frac{W(1+N^2)}{4} - 2P\right];$$

$$l_1 = \frac{H_1}{h},\quad l_2 = \frac{H_2}{h},\quad \overline{T}_2 = \frac{T_2 - T_W}{V^2\mu^N}\lambda_l.$$

As in Ref. 1, let $\lambda_s/\lambda_l = 78$ and $l_1 = l_2 = l = 100$. In Figs. 2 and 3 we present temperature profiles for the flow of an MPL with different values of the boundary-condition parameter ξ in the same channel (Fig. 2) and in channels of different size h with $\xi = 1$ (Fig. 3).

As seen from the figures, allowance for the internal microstructure of the liquid when calculating the temperature in the channel always results in its increase. This result is the direct opposite of that obtained in [1] for Poiseuille flow, where allowance for intrinsic particle rotation results in a decrease in $\overline{T}_2(\tilde{y})$, but is not at variance with it at all. The point is that the character of the assignment of the "cause" resulting in MPL flow is different in the first part [1] and the part of the present article under consideration here.

In Poiseuille flow [1] it was more appropriate to consider a source of motion with a dynamic character — a pressure drop (rather than an assigned volumetric flow rate, for example). In theoretically investigating the flow of a liquid with a specific microstructure

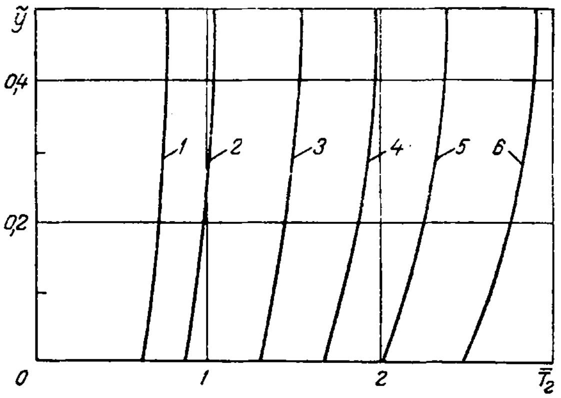

Fig. 4. Temperature field in a channel with k = 1 and ξ = 0.9 for δ = 0 (1); 1 (2); 3 (30; 5 (4); 7 (5); 10 (6).

under a given pressure drop within the framework of the models of a micropolar and a Newtonian liquid, in the latter case under certain conditions in which the micropolarity of the medium is noticeably manifested, we use an overstated value of the flow velocity, since the tabular value of the coefficient of shear viscosity of a given liquid is less than its actual "equivalent" viscosity. Therefore, when a liquid with a microstructure is treated as a Newtonian liquid, the calculated values of $\overline{T}_2(\tilde{y})$ prove to be overstated.

In an investigation of Couette flows one assigns a "cause" of motion with a kinematic character: the relative velocity of motion of the channel walls. The viscous-energy-dissipation function, calculated within the framework of the model of a Newtonian liquid, is determined by the velocity gradient. Allowance for the internal microstructure of the liquid in the calculation of the velocity gradient does not result in its great variation in an average over the cross section (see Fig. 1a). However, microrotation does make its contribution to the value of the viscous-dissipation function (6). Therefore, the dissipative heating calculated with allowance for the intrinsic rotation of liquid particles is greater than that found within the framework of the model of a Newtonian liquid.

The quantity $\overline{T}_2(\tilde{y})$ increases as h decreases and ξ grows. As k → 0,

$$\frac{\overline{T}_2(\tilde{y})}{\overline{T}_2^N(\tilde{y})} \to 1 + \frac{\delta_0 \xi^2}{2},$$

where $\overline{T}_2^N(\tilde{y})$ corresponds to a calculation without allowance for the internal microstructure (curve 1 of Fig. 3). Even with k = 0.1 a decrease in k results in hardly any change in the temperature profile. Curves 2 and 4 in Fig. 3 correspond to two of the possible combinations of the parameters δ_0 and ξ for water, for which $\delta_0 \xi$ = 1.45 [1].

In Fig. 4 we present curves of $\overline{T}_2(\tilde{y})$ corresponding to flows of MPL with different microstructure characteristics in channels with different transverse sizes h. With δ_0 = 10, k = 1, and ξ = 0.9, for example, allowance for the internal microstructure of the liquid when calculating the temperature in the channel results in an almost fourfold increase in it.

The velocity of Couette flows is very high in practice, in the flow of a lubricant, for example. In this case the temperature increase of the liquid due to viscous energy dissipation can be very pronounced. In such cases, such as in the determination of the optimum modes of operation of lubricating devices, it can prove necessary to allow for the intrinsic rotation of lubricant particles when estimating the influence of dissipative heating on its properties.

NOTATION

h, distance between the plates; H_1, H_2, thicknesses of the moving and fixed plates, respectively; V, velocity of plate motion; v_x, ν_z, components of velocity and microrotation vectors; κ, μ, γ, material constants of the micropolar liquid; α, parameter of the boundary conditions; Φ, viscous-energy-dissipation function; λ_s, λ_l, coefficients of thermal conductivity of wall material and liquid; T, temperature; T_w, temperature of outer surfaces of channel; $\vec{w}$ = (1/2)rot $\vec{v}$, vorticity vector.

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FLUCTUATIONS AND TRANSPORT IN AN ELECTRIC FIELD

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It is shown that thermodynamic fluctuations in a liquid in an applied electric field generate microconvective motion causing mass and heat dispersion. The corresponding dispersion coefficients can be comparable in value or even exceed the coefficients of molecular diffusion and thermal conductivity.

An applied electric field can significantly stimulate heat- and mass-transport processes in liquids with very different electric and magnetic properties [1-3]. The usual interpretation is that this is due to the appearance of specific convective motions in the liquid from Coulomb pondermotive forces and Lorentz forces and also from convective transport of volume charge.

However, these effects do not exhaust all possible effects from the field, since many examples are known where heat and mass exchange are stimulated in situations where convection does not appear. In addition, the necessary condition for the appearance of electro or magnetohydrodynamic convection is that there be nonuniformities in either the properties of the liquid or the external field. Stimulation of heat and mass transport in an applied field is observed in conditions when both the liquid and the field can certainly be considered as uniform. Therefore, besides convection, one concludes that there is another fundamental effect of the electric field on transport processes, in general not involving the violation of mechanical stability of the liquid.

It is shown below that the latter effect comes from the appearance of additional microconvective dispersion in a liquid which is macroscopically at rest. This dispersion is due to random small-scale fluctuations which appear due to the interaction of the external field with random fluctuations of the volume charge. The latter is in turn caused by the usual thermodynamic fluctuations. Since the purpose of the present paper is to demonstrate the existence of this effect, we consider only the simplest examples in a uniform applied electric field and with several simplifying assumptions.

Fluctuations and Dispersion. At small Reynolds numbers the equations of hydrodynamics in the presence of the pondermotive force are given by

$$\gamma \partial \mathbf{v}/\partial t = -\nabla p + \mu \Delta \mathbf{v} + \rho \mathrm{E}, \ \mathrm{div}\mathbf{v} = 0. \quad (1)$$

As in [4], the above expression for the pondermotive force is also assumed to be valid for a conducting liquid if the conductivity can be made as small as desired. The unperturbed state is the state of rest where $\mathbf{v} = 0$, $p = \text{const}$, $\rho = 0$, but $\mathbf{E} = \mathbf{E}_0 \neq 0$.

The theory of hydrodynamic fluctuations [5] reduces to (1) and the general equation of heat transport, where the variables are regarded as small fluctuations and local random stresses and heat fluxes are added to the equations. It is then not difficult to obtain directly equations for the correlation functions [6]. Here we will assume that the hydrodynamic fluctuations are generated mainly by fluctuations in the pondermotive force, so that we do not need to introduce additional random terms in (1). The spectral properties of the fluctuations are studied with the help of the correlation theory of stationary random processes [7]: any random function f of the coordinates $\mathbf{r}$ and time t is represented as a Fourier–Stieltjes integral with random measure dZ_f:

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